7. In all of the languages considered in this exercise, R is a binary relation symbol, $*$ and $\oplus$ are binary function symbols, c and d are constant symbols.

We will write $x \oplus y$ and $x * y$ respectively, rather than $\oplus xy$ and $*xy$ (with a reminder that this necessitates the use of parentheses when writing terms). x^2 will be an abbreviation for $x * x$.

(a) In each of the following six cases ($1 \leq i \leq 6$), a language L_i and two L_i-structures $\mathcal{A}_i$ and $\mathcal{B}_i$ are given and you are asked to find a closed formula of L_i that is true in $\mathcal{A}_i$ and false in $\mathcal{B}_i$.

(1) $L_1 = \{R\}$	$\mathcal{A}_1 = \langle \mathbb{N}, \leq \rangle$	$\mathcal{B}_1 = \langle \mathbb{Z}, \leq \rangle$
(2) $L_2 = \{R\}$	$\mathcal{A}_2 = \langle \mathbb{Q}, \leq \rangle$	$\mathcal{B}_2 = \langle \mathbb{Z}, \leq \rangle$
(3) $L_3 = \{*\}$	$\mathcal{A}_3 = \langle \mathbb{N}, \times \rangle$	$\mathcal{B}_3 = \langle \wp(\mathbb{N}), \cap \rangle$
(4) $L_4 = \{c, *\}$	$\mathcal{A}_4 = \langle \mathbb{N}, \mathbf{1}, \times \rangle$	$\mathcal{B}_4 = \langle \mathbb{Z}, \mathbf{1}, \times \rangle$
(5) $L_5 = \{c, d, \oplus, *\}$	$\mathcal{A}_5 = \langle \mathbb{R}, \mathbf{0}, \mathbf{1}, +, \times \rangle$	$\mathcal{B}_5 = \langle \mathbb{Q}, \mathbf{0}, \mathbf{1}, +, \times \rangle$
(6) $L_6 = \{R\}$	$\mathcal{A}_6 = \langle \mathbb{Z}, \equiv_2 \rangle$	$\mathcal{B}_6 = \langle \mathbb{Z}, \equiv_3 \rangle$

($\times$ and $+$ are the usual operations of multiplication and addition, $\cap$ is the operation of intersection, $\equiv_p$ is the relation of congruence modulo p.)

(b) For each of the following closed formulas of the language $\{c, \oplus, *, R\}$, find a model of the formula as well as a model of its negation.

$$F_1: \quad \forall u \forall v \exists x (\neg v \simeq c \Rightarrow u \oplus (v * x) \simeq c)$$

$$F_2: \quad \forall u \forall v \forall w \exists x (\neg w \simeq c \Rightarrow u \oplus (v * x) \oplus (w * x^2) \simeq c)$$